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APPROXIMATE TECHNIQUE FOR EVALUATING THE MOYER INTEGRAL IN THE MOYER MODEL OF HADRON SHIELDING

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RADIATION PHYSICS NOTE 117

J. Donald Cossairt

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INTRODUCTION

In the design of accelerator radiological shielding, practitioners at Fermilab and elsewhere often used the so-called Moyer Model. The general methodology employed in this semiempirical model has been discussed in the literature and reviewed in detail by Cossairt[1]. The purpose of this note is to suggest a methodology for simplifying such calculations in a form that makes them more amenable to the employment of spreadsheets when using the Moyer Model for a line source. To assure adequate definition of variables, this note begins from a statement of the Moyer Model for a point source and proceeds to the specifics of concern here.

The Moyer model for Point Source

Figure 1 shows the "geometry" involved in the Moyer Model as applied to a point source and defines the coordinates. In this figure, the proton beam is assumed to interact at a point such that some neutron radiation exposure is incurred at a point, O, external to a composite shield. Each layer of such a shield, of which three are shown in Fig. 1, has a thickness x_i.

The parameter ζ is introduced to take care of the multiple (n) shielding components:

$$\zeta = \sum_{i=1}^{n} \frac{x_i}{\lambda_i} \equiv \sum_{n=1}^{n} \zeta_i \qquad (1)$$

where the sum is over the i layers of shielding and ζ_i is the thickness of each layer in units of effective interaction length.

Obviously, the values of thickness x_i and effective interaction lengths λ_i have to be in consistent units. As is seen below, linear dimensions (e.g., cm) are preferable to areal densities (e.g., g/cm^2) for these two quantities.

Moyer introduced the following simplifying assumptions in developing this model:

A. $\lambda(E) = \lambda$ = constant for $E \geq 150$ MeV, and $\lambda(E) = 0$ for $E < 150$ MeV. This is a simplified rendering of the leveling-off of the inelastic cross section at high energy and, in effect results in "ignoring" the neutrons with $E < 150$ MeV as far as the propagation of the cascade is concerned.

B. The flux density of neutrons emitted at angle θ can be represented by a simple function, independent of incident particle energy, multiplied by a multiplicity factor that depends only on the incident energy. This factorization is the result of a number of studies that suggest that the angular dependence is independent of the incident energy.

C. The fluence to dose equivalent conversion factor for neutrons with energy > 150 MeV, P_{150}, is not strongly dependent on energy. Thus the dose equivalent just outside of the shield due to those neutrons with kinetic energies > 150 MeV is given by:

$$H(E_n > 150 \text{ MeV}) = P_{150}\, \phi(E_n > 150 \text{ MeV}) \tag{1}$$

where $\phi(En > 150$ MeV) is the fluence of neutrons exceeding 150 MeV in kinetic energy.

In effect, a "fudge factor", k, is being used to account for the dose equivalent due to the low energy neutrons. The total dose equivalent, H, then is given by

$$H = kH(E_n > 150 \text{ MeV}) \quad \text{where } k \geq 1. \tag{2}$$

This implicitly assumes that the low-energy neutrons are in equilibrium with those > 150 MeV so that the spectrum no longer changes with depth. This is a valid assumption for a shield more than a few, probably about two, mean free paths thick.

Recent work, notably that of Stevenson, et. al[2]., Thomas and Stevenson[3], and Thomas and Thomas[4], has determined that the data indicate that the angular factor, denoted $f(\theta)$, is given by:

$$f(\theta) = \exp(-\beta\theta), \tag{3}$$

and that, in fact, $\beta \approx 2.3$ radians^{-1} (for $E_n > 150$ MeV) provides the best fit to data.

Thus, it turns out that:

$$H = \frac{H_o(E_p)\exp(-\beta\theta)\exp\{-\zeta \csc(\theta)\}}{(r \csc(\theta))^2} \tag{4}$$

where, as shown in Fig. 1 $\quad r = R + \sum_{i=1}^{n} x_i$ and $\quad (5)$

where $H_o(E_p)\exp(-\beta\theta)$ is determined from yield data and empirical measurements. $H_o(E_p)$ is best fit as a power law; $H_o(E_p) = kE^n$. From such results (per proton which interacts at a point):

$$H_o(E_p) = [(2.84 \pm 0.14) \text{ X } 10^{-13}]\, E_p^{(0.80 \pm 0.10)} \text{ Sv m}^2. \tag{6}$$

$$= 2.84 \text{ X } 10^{-8}\, E_p^{0.8} \text{ mrem m}^2 = 2.8 \text{ X } 10^{-4}\, E_p^{0.8} \text{ mrem cm}^2$$

with E_p in GeV (per proton). These results are derived for relatively "thick" targets (like accelerator magnets) in tunnel geometries. Reference 5, based on Monte-Carlo results gives values for "thin" targets of $k = 2.0 \text{ X } 10^{-13}$ (Sv) and $n = 0.5$. A beam pipe would be an example of a "thin" target. The differences thus reflect buildup in the shower. Also from Ref. 5, for thick lateral shields where the cascade immediately becomes fully developed, and self-shielding is evident, $k = (6.9 \pm 0.1) \text{X } 10^{-15}$ (Sv) independent of target material.

Recommended values of λ are,

concrete: 1170 ± 20 kg/m^2 = 117 g/cm^2

others: $428A^{1/3}$ kg/m^2 = $42.8A^{1/3}$ g/cm^2, (7)

where A is the atomic mass number.

These values are 15-30% larger than the "nuclear interaction lengths" and are reflective of the shower phenomena discussed above. For compounds, it is suggested that the mass-weighted mean of the atomic mass number should be used.

The Moyer Model for Line Sources

Line sources can be important even though most large accelerators would not be operated in a manner in which the whole accelerator, or extracted beam line would constitute such a source. McCaslin, et. al.[6] (repeated in Refs. 1 and 5) has demonstrated that in practical circumstances, rather short lengths of localized beam loss, typically less than 10 meters, can have the same effect as an "infinite" line source. Also, the employment of the line source model can be important when one wants to considered the long-term effects of "random" losses of beam over the entirety of an accelerator or beam line for extended periods of time. One can derive the Moyer Model for a line source by integrating the effects of uniform sources of one proton interacting per unit length as expressed by Eq. (4). In this integration, the dose equivalent from the individual increments along the line source contribute to the total at any given point, P, external to the shield. Figure 2 shows the integration variables.

$$H = H_o(E_p)\int_{-\infty}^{\infty} d\ell \, \frac{\exp(-\beta\theta)\exp\{-\zeta\csc(\theta)\}}{r^2\csc^2(\theta)} =$$

$$H_o(E_p)\int_0^{\pi} d\theta \, r\csc^2(\theta)\,\frac{\exp(-\beta\theta)\exp\{-\zeta\csc(\theta)\}}{r^2\csc^2(\theta)} =$$

$$\frac{H_o(E_p)}{r}\int_0^{\pi} d\theta \exp(-\beta\theta)\exp\{-\zeta\csc(\theta)\} = \frac{H_o(E_p)}{r}M(\beta,\zeta) \qquad (8)$$

(per interacting proton, or proton "lost", per unit length).

The integral in the above, $M(\beta,\zeta)$, is the so-called Moyer integral. The values of this integral have been tabulated by Routti and Thomas[7]. Unfortunately, this integral is of a form requiring numerical integration. In view of the above results, $M(2.3,\zeta)$ has obvious special significance. Tesch[8] has made an important contribution in that he determined an approximation to this integral which others have come to call the "Tesch approximation":

$$M_T(2.3,\zeta) = 0.065\exp(-1.09\zeta). \qquad (9)$$

For "intermediate" values of ζ (i.e., $2 \leq \zeta \leq 15$), $M_T(2.3,\zeta)$ can be used instead of $M(2.3,\zeta)$ to simplify calculations. As is pointed out in Refs. 5 and 6, the agreement is satisfactory over a significant domain of ζ, but is not so good for both relatively small and relatively large values of this parameter. The ratio, $M(2.3,\zeta)/M_T(2.3,\zeta)$ has been plotted in Fig. 3. This ratio has been fit by a ninth-order polynomial which was determined to be:

$$M(2.3,\zeta)/M_T(2.3,\zeta)_{fit} = 4.95030852628238 - 5.58842754668814\zeta + 3.44285488235745\zeta^2$$

$$- 1.13781132219873\zeta^3 + 0.219559064481144\zeta^4 - 0.0258865822707814\zeta^5$$

$$+ 0.00188805741245252\zeta^6 - 0.0000830573335841856\zeta^7$$

$$+ 2.01811879781487\text{E-}06\zeta^8 - 2.07907070756853\text{E-}08\zeta^9. \qquad (10)$$

The polynomial fit is also displayed in Fig. 3. It is important to note that erroneous results can be obtained if the full complement of significant figures is not used in a calculation. Fig. 4 shows the deviations of $M(2.3,\zeta)/M_T(2.3,\zeta)_{fit}$ from a true representation of the value of $M(2.3,\zeta)/M_T(2.3,\zeta)$ by plotting the ratio of these two quantities. This fit is only valid over the domain considered, $0 \leq \zeta \leq 20$. Outside of this domain, the value of the polynomial will approach infinite values. It is seen that within the stated domain, the agreement is easily good to within 10 per cent. This error is certainly less than other errors associated with the use of the Moyer Model, especially for values of ζ less than approximately two, where the shower is not well-developed.

Thus, in the context of the Moyer Model one can determine the effect of a line source by approximating the value of $M(2.3,\zeta)$ in Eq. (8) as follows:

$$M(2.3,\zeta) \approx \{M(2.3,\zeta)/M_T(2.3,\zeta)_{fit}\}\, M_T(2.3,\zeta). \qquad (11)$$

Thus, one can use the equation directly in spreadsheets to rather accurately give a value of the Moyer Integral.

EXAMPLE OF SUCH A SPREADSHEET

Table 1 displays a Microsoft Excel™ spreadsheet incorporating this approximation to the Moyer Model for line sources. Table 2 displays the formulae set up on the spreadsheet. Both of these Tables have hidden rows under the headings "r(m), zeta,..." in order to save space. At the top of the spreadsheet, one enters the proton (or hadron) kinetic energy in GeV and the thicknesses of each of the layers of a three layer shield in meters, working outward. The densities are entered in g/cm^3 and the interaction lengths, in g/cm^2, are entered according to Eq. (7). From these thicknesses, the spreadsheet calculates the value of ζ_i for each of the three layers. Below the entry of the beam energy, the spreadsheet calculates the value of $H_0(E_p)$ according to Eq. (6). Below that calculation, one needs to enter the inner radius of the tunnel, R (meters), and the number of protons estimated to be lost per meter for some time interval of concern. Below these entries, the spreadsheet calculates the sum of the shield thickness in meters and the value of r_{max}, the outer boundary of the problem in meters.

The particular example is for the case of 10^{11} 800 GeV protons per meter lost on accelerator magnets in a tunnel shielded by three layers; the first of concrete, the second of iron, and the third of earth. One wants to select the thickness of the outer shield that will attenuate the radiation due to these interactions down to some desired dose equivalent. The entry "Description of Setup" is intended to provide the user with a place to note comments on the particular calculation involved. Below this point, the spreadsheet is set up to calculate the effect of various thicknesses of the outer shield starting with the outer boundary of the second layer and incrementing through some range of outer shielding material. In the spreadsheet as printed, the increment was 0.2 meters. This could be modified to some different value in actual use of the method by modifying the spreadsheet. For each value of r (column A), the total value of thickness ζ included by the corresponding value of r, in units of effective interaction lengths, is calculated (column B). In column C, the Moyer integral $M(2.3,\zeta)$ is calculated using Eq. (11). In column D, the dose equivalent per interacting proton per meter is calculated using Eq. (8) while in column E the dose equivalent due to the beam loss per meter specified in entry C6 is given. At the bottom of the spreadsheet, the coefficients of the polynomial used in Eq. (10) are set forth.

Using this spreadsheet to identify values of dose equivalent over a range of thickness of the third layer is implicitly making the assumption that backscatter from outer layers is negligible. Backscatter, of course, is not considered in the Moyer Model and has generally been determined to be insignificant for shields of reasonable thickness using Monte-Carlo simulations

Finally, if one desires to consider a shield of only one or two materials, one can simply set the thickness of the first two materials (or the first material) to zero and just work with the third (outer) material.

ACKNOWLEDGMENT

The author would like to thank Kamran Vaziri and Alex Elwyn for helpful comments concerning both this paper and also R. P. Note 118.

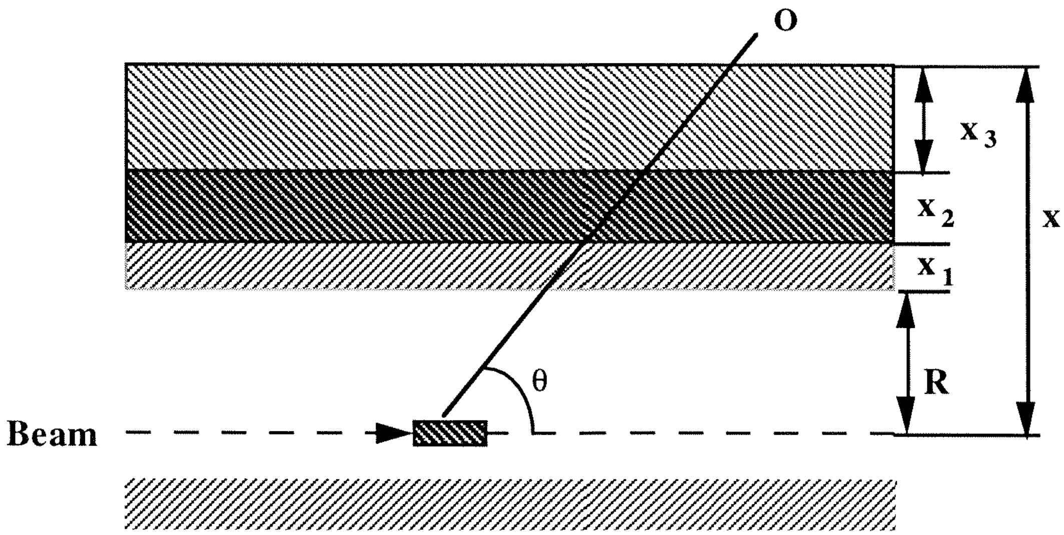

Figure 1 Geometry of the Moyer Model as applied to a point source for a composite shield consisting of three layers.

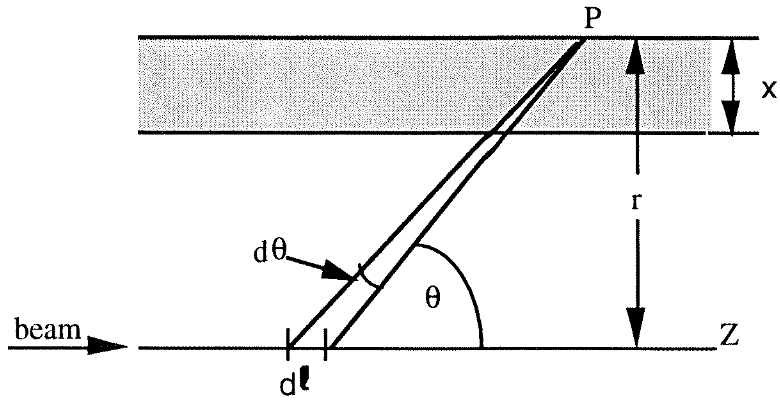

Figure 2 Integration variables for deriving the Moyer Model for a line source. The shielding material of thickness x could well be a composite shield as treated by Eq. (1). The Z-axis respresents a string of accelerator magnets.

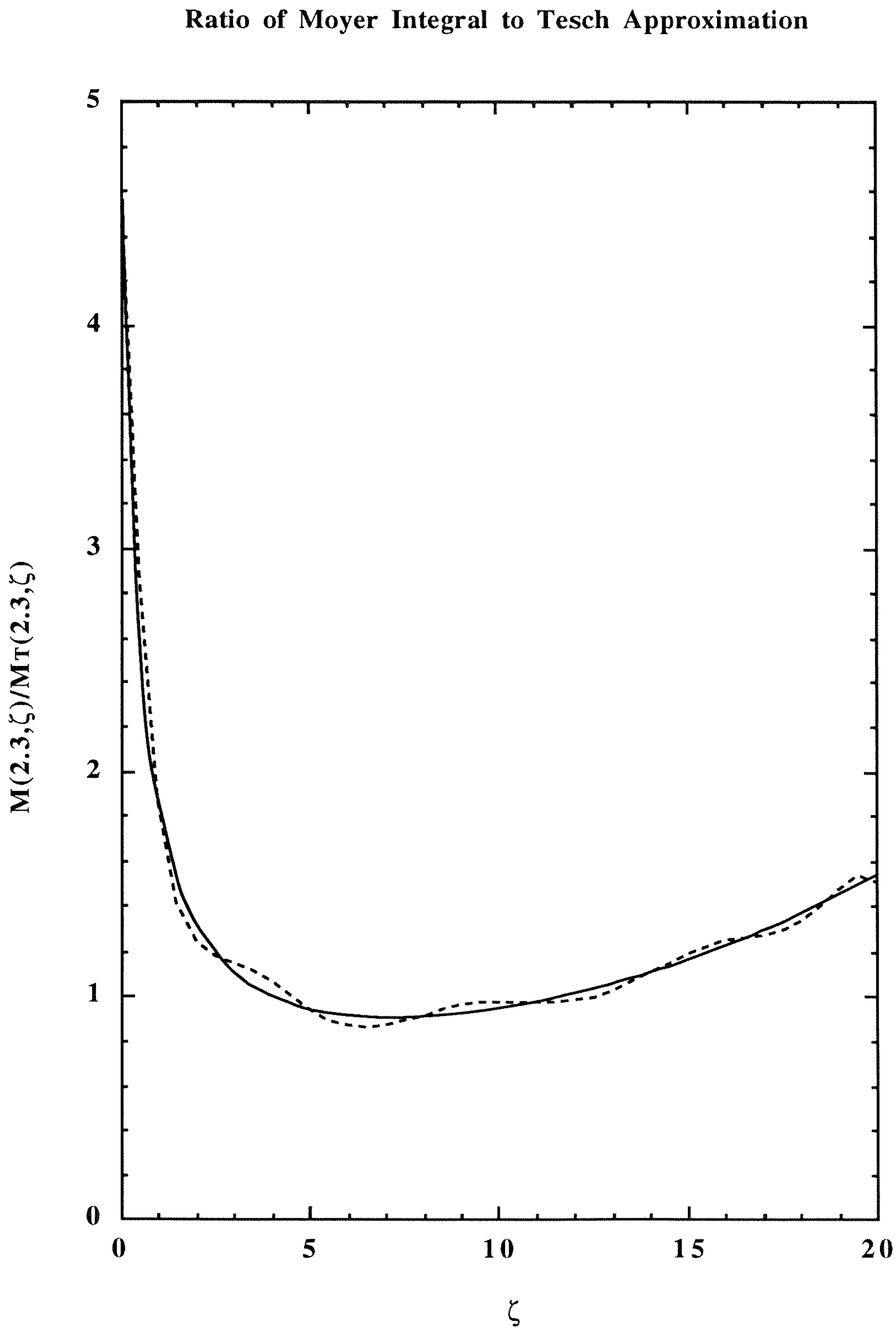

Figure 3 The solid curve is a plot of the ratio of the Moyer Integral $M(2.3,\zeta)$ to the Tesch approximation, $M_T(2.3,\zeta)$ as a function of ζ. The dashed curve is the result of using a ninth-order polynomial to fit this ratio.

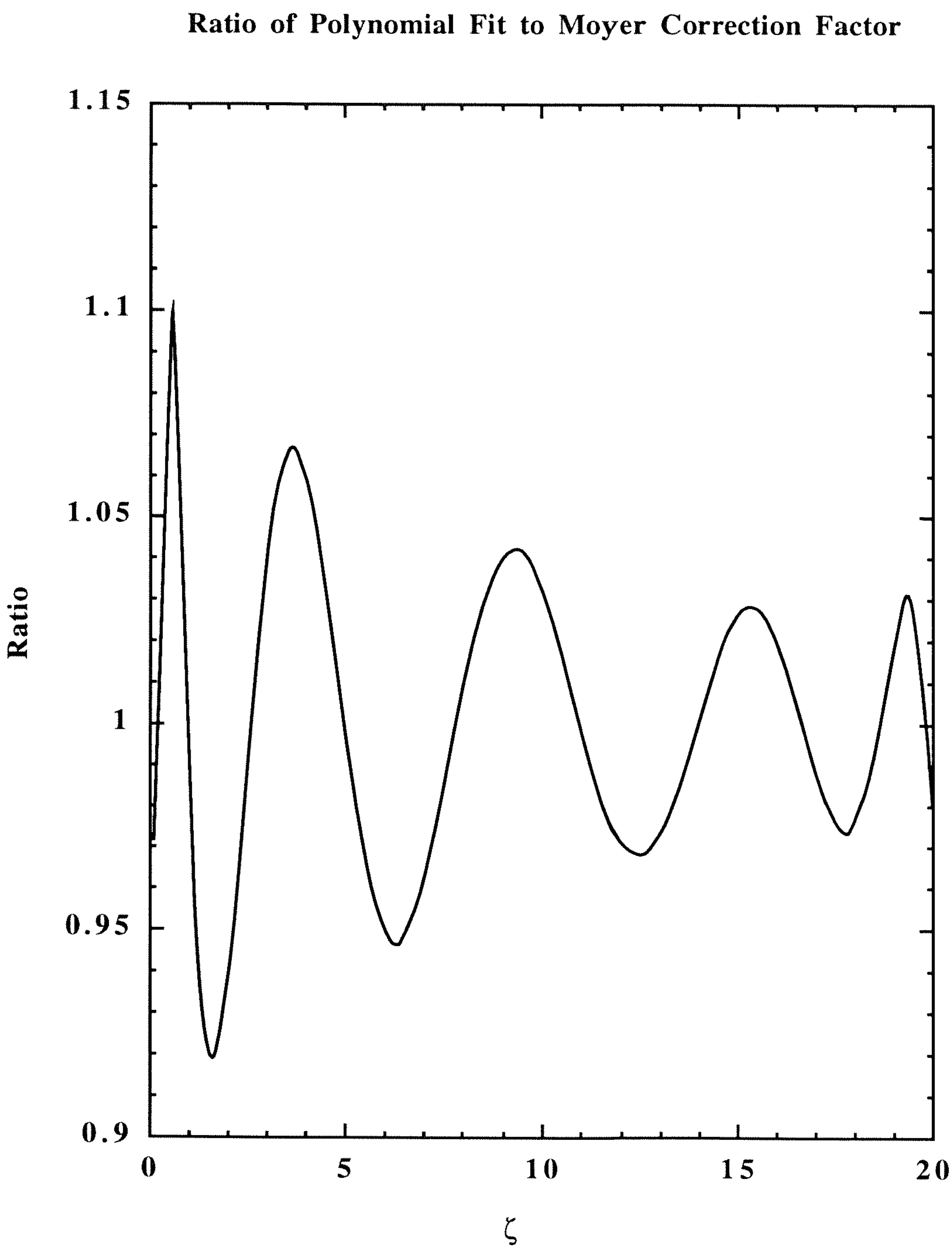

Figure 4 Plot of the ratio of $M(2.3,\zeta)/M_T(2.3,\zeta)_{fit}$ to the true value of $M(2.3,\zeta)/M_T(2.3,\zeta)$. A perfect representation of the latter quantity would be a straight horizontal line with an abscissa of unit value.

Table 1 Output of a Microsoft Excel™ spreadsheet employing formulae developed in this work. Some rows have been "hidden" in order to minimize boredom and save space.

	A	B	C	D	E	F	G	H	I	J	K
1	PARAMETERS					(meters)	rho	layer thick	lambda		(d/lambda)
2							(g/cm^3)	(g/cm^2)	(g/cm^2)		
3	*Enter Energy->*	Ep = (GeV)	800.00	*Enter layer 1 (X1) (m) ->*		0.3000	2.4	72.00	117	zeta1 =	0.615
4		Ho(Ep) (mrem m^2) =	5.97E-06	*Enter layer 2 (X2) (m) ->*		0.6000	7.5	450.00	163.7	zeta2 =	2.749
5	*Enter tun. rad (m) ->*	R (inner boundary, m)	1.50	*Enter layer 3 (X3) (m) ->*		6.0000	2.25	1350.00	117	zeta3 =	11.538
6	*Enter proton loss/m*	Beam/meter	1.00E+11								
7					sum= (x)	6.900	(meters)	rmax =	8.400	(meters)	
8	DESCRIPTION OF SETUP: Three layer shield, concrete followed by iron followed by earth.										
9											
10	r (m)	zeta	Moyer(2.3,zeta)	H (mrem/	H (mrem)						
11				proton/meter)							
12											
13	2.400	3.364	1.87E-03	4.65E-09	4.65E+02						
14	2.600	3.749	1.19E-03	2.73E-09	2.73E+02						
15	2.800	4.134	7.51E-04	1.60E-09	1.60E+02						
37	7.200	12.595	7.11E-08	5.90E-14	5.90E-03						
38	7.400	12.980	4.78E-08	3.85E-14	3.85E-03						
39	7.600	13.364	3.22E-08	2.53E-14	2.53E-03						
40	7.800	13.749	2.18E-08	1.67E-14	1.67E-03						
41	8.000	14.134	1.48E-08	1.10E-14	1.10E-03						
42	8.200	14.518	1.00E-08	7.30E-15	7.30E-04						
43	8.400	14.903	6.78E-09	4.82E-15	4.82E-04						
44	8.600	15.287	4.56E-09	3.17E-15	3.17E-04						
45	8.800	15.672	3.05E-09	2.07E-15	2.07E-04						
46	9.000	16.057	2.03E-09	1.35E-15	1.35E-04						
47	9.200	16.441	1.35E-09	8.76E-16	8.76E-05						
48	9.400	16.826	8.94E-10	5.68E-16	5.68E-05						
49	9.600	17.210	5.93E-10	3.69E-16	3.69E-05						
50	9.800	17.595	3.96E-10	2.41E-16	2.41E-05						
51	10.000	17.980	2.66E-10	1.59E-16	1.59E-05						
52											
53											
54		Coeff. of Polynomial:									
55		_a =	4.950308526282380								
56		_b =	-5.588427546688140								
57		_c =	3.442854882357450								
58		_d =	-1.137811322198730								
59		_e =	2.19559064481144E-01								
60		_f =	-2.58865822707814E-02								
61		_g =	1.88805741245252E-03								
62		_h =	-8.30573335841856E-05								
63		_i =	2.01811879781487E-06								
64		_j =	-2.07907070756853E-08								

Table 2-page 1 Same Microsoft Excel™ spreadsheet as in Table 1 only showing formulae. Some rows have been "hidden" in order to minimize boredom and save space.

	A	B
1	**PARAMETERS**	
2		
3	*Enter Energy->*	Ep = (GeV)
4		Ho(Ep) (mrem m^2) =
5	*Enter tun. rad (m) ->*	R (inner boundary, m) = [R_o]
6	*Enter proton loss/m*	Beam/meter [Beam]
7		
8	**DESCRIPTION OF SETUP: Three layer shield, co**	
9		
10	r (m)	**zeta**
11		
12		
13	=C5+_t1+_t2	=((A13-R_o-_t1-_t2)*100*rho3/lambda3)+zeta1+zeta2
14	=A13+0.2	=((A14-R_o-_t1-_t2)*100*rho3/lambda3)+zeta1+zeta2
15	=A14+0.2	=((A15-R_o-_t1-_t2)*100*rho3/lambda3)+zeta1+zeta2
48	=A47+0.2	=((A48-R_o-_t1-_t2)*100*rho3/lambda3)+zeta1+zeta2
49	=A48+0.2	=((A49-R_o-_t1-_t2)*100*rho3/lambda3)+zeta1+zeta2
50	=A49+0.2	=((A50-R_o-_t1-_t2)*100*rho3/lambda3)+zeta1+zeta2
51	=A50+0.2	=((A51-R_o-_t1-_t2)*100*rho3/lambda3)+zeta1+zeta2
52		
53		
54		**Coeff. of Polynomial:**
55		_a =
56		_b =
57		_c =
58		_d =
59		_e =
60		_f =
61		_g =
62		_h =
63		_i =
64		_j =

Table 2-page 2 Same Microsoft Excel™ spreadsheet as in Table 1 only showing formulae. Some rows have been "hidden" in order to minimize boredom and save space.

	C	D
1		
2		
3	800	*Enter layer 1 (X1) (m) ->*
4	=(0.0000000284)*C3^0.8	*Enter layer 2 (X2) (m) ->*
5	1.5	*Enter layer 3 (X3) (m) ->*
6	100000000000	
7		
8		
9		
10	**Moyer(2.3,zeta)**	H (mrem/
11		proton/meter)
12		
13	=(_a +_b*B13 + _c*B13^2 +_d*B13^3 + _e*B13^4+_f*B13^5 + _g*B13^6+_h*B13^7 + _i*B13^8+_j*B13^9)*0.065*EXP(-1.09*B13)	=HoEp*C13/(A13)
14	=(_a +_b*B14 + _c*B14^2 +_d*B14^3 + _e*B14^4+_f*B14^5 + _g*B14^6+_h*B14^7 + _i*B14^8+_j*B14^9)*0.065*EXP(-1.09*B14)	=HoEp*C14/(A14)
15	=(_a +_b*B15 + _c*B15^2 +_d*B15^3 + _e*B15^4+_f*B15^5 + _g*B15^6+_h*B15^7 + _i*B15^8+_j*B15^9)*0.065*EXP(-1.09*B15)	=HoEp*C15/(A15)
48	=(_a +_b*B48 + _c*B48^2 +_d*B48^3 + _e*B48^4+_f*B48^5 + _g*B48^6+_h*B48^7 + _i*B48^8+_j*B48^9)*0.065*EXP(-1.09*B48)	=HoEp*C48/(A48)
49	=(_a +_b*B49 + _c*B49^2 +_d*B49^3 + _e*B49^4+_f*B49^5 + _g*B49^6+_h*B49^7 + _i*B49^8+_j*B49^9)*0.065*EXP(-1.09*B49)	=HoEp*C49/(A49)
50	=(_a +_b*B50 + _c*B50^2 +_d*B50^3 + _e*B50^4+_f*B50^5 + _g*B50^6+_h*B50^7 + _i*B50^8+_j*B50^9)*0.065*EXP(-1.09*B50)	=HoEp*C50/(A50)
51	=(_a +_b*B51 + _c*B51^2 +_d*B51^3 + _e*B51^4+_f*B51^5 + _g*B51^6+_h*B51^7 + _i*B51^8+_j*B51^9)*0.065*EXP(-1.09*B51)	=HoEp*C51/(A51)
52		
53		
54		
55	4.95030852628238	
56	-5.58842754668814	
57	3.44285488235745	
58	-1.13781132219873	
59	0.219559064481144	
60	-0.0258865822707814	
61	0.00188805741245252	
62	-0.0000830573335841856	
63	2.01811879781487E-06	
64	-2.07907070756853E-08	

Table 2-page 3 Same Microsoft Excel™ spreadsheet as in Table 1 only showing formulae. Some rows have been "hidden" in order to minimize boredom and save space.

	E	F	G	H	I	J	K	L
1		(meters)	rho	layer thick	lambda		(d/lambda)	
2			(g/cm^3)	(g/cm^2)	(g/cm^2)			
3	[_t1]	0.3	2.4	=G3*F3*100	117	zeta1 =	=H3/I3	
4	[_t2]	0.6	7.5	=G4*F4*100	163.7	zeta2 =	=H4/I4	
5	[_t3]	6	2.25	=G5*F5*100	117	zeta3 =	=H5/I5	
6								
7	sum= (x)	=SUM(F3:F5)	(meters)	rmax =	=F7+C5	(meters)		
8								
9								
10	H (mrem)							
11								
12								
13	=D13*Beam							
14	=D14*Beam							
15	=D15*Beam							
48	=D48*Beam							
49	=D49*Beam							
50	=D50*Beam							
51	=D51*Beam							
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